Adaptive Controller using Fuzzy Modeling and Sigmoid Generated Fixed Point Transformation

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***Abstract*—The great majority of the adaptive nonlinear control design are based on Lyapunov's 2nd or commonly referred to as the Direct method. In the last years the " *Sigmoid Generated Fixed Point Transformation (SGFPT)*" has been introduced for replacing the Lyapunov technique. This systematic method has been proposed for the generation of whole families of Fixed Point Transformations. In addition it has been extended from Single Input Single Output (SISO) to Multiple Input Multiple Output (MIMO) systems. Recently, several model building issues are increasingly replaced by soft-computing based methods. In spite of the classical hard-computing methods the intelligent methodologies are able to deal with imprecisions, uncertainties, etc. by an efficient and robust way. Fuzzy logic is widely used for modeling complex and ill-defined systems. In this paper we apply the fuzzy modeling in the SGFPT control design. The applicability of the proposed scheme is confirmed by the adaptive control of the inverted pendulum system. In the investigations an *"affine"*, and a *"soft computing-based"* model were compared. Simulation results validate that the presented technique fulfills the performance criteria.**

***Keywords*—Adaptive Control; Fixed Point Transformation; Fuzzy Modeling; Robust Fixed Point Transformation (RFPT); Iterative Learning Control; Banach's Fixed Point Theorem; Sigmoid Generated Fixed Point Transformation (SGFPT)**

I. INTRODUCTION

In adaptive nonlinear control Lyapunov's *direct* method (or also called the *second method* of Lyapunov) have become a very efficient tool because it does not require to analytically solve the equations of motion [1] [2]. Many researches on the applications and extensions of the Lyapunov-function have been reported. In spite of the simplicity of the geometric interpretation of this technique, the application of Lyapunov's second method in engineering practice is far difficult due to its high computational need. On the other hand, finding the appropriate Lyapunov function candidate and also the automation of such algorithms, etc., are very complicated tasks. A great majority of control design techniques are based on Lyapunov's theorem. For instance, the *Model Reference Adaptive Controller (MRAC)* applies it for tuning the feedback parameters [3]–[6], while the *Slotine-Li Adaptive Robot Controller (SLARC)* and also the *Adaptive Inverse Dynamics Robot Controller (AIDRC)* control strategies use Lyapunov's second method for tuning the parameters of the analytical system model [7]. A summary on the application of Lyapunov's second method that highlights the relating practical issues can be found in [8]. The Robust Fixed Point Transformation (RFPT)-based control method has been introduced for replacing Lyapunov's direct method, has revealed in [9]–[11]. The core theory the Fixed Point Transformation-based control approach relies on the transformation of the control signal's calculation problem into the task of finding an appropriate fixed point of a contractive map [8]. Using this contractive map the fixed point can be derived by generating an iterative sequence. The other specific feature of the RFPT-based control method, that it strictly separates the "kinematic" and "dynamic" aspects of the control task and predefines the "desired system response" by the use of purely kinematic terms. In addition, it assumes the existence of the approximate dynamic model of the system under control that is used in the estimation of the necessary dynamic terms that are required for the realization of the kinematic specifications. The undesired effects of modeling imprecisions and unknown external disturbances are compensated by the adaptive deformation of the *desired response* r^{Des} as the input of the approximate model. By this deformation, the desired response can be obtained as the realized one. The adaptive deformation is carried out by some kinematically expressed trajectory error reduction, i.e. by the comparison of the *desired response* r^{Des} and the *actually observed response* r^{Act}. The *actually observed response* is deformed due to the exact system under control, formally $r^{Act} = f(r^{Des}, \dots)$ where the symbol "…" stands for the unknown state variables, for e.g. coupled subsystems, disturbances, etc. Due to the additive noises, etc.,

$r^{Act} \neq r^{Des}$. Hence, the input of the response function from r^{Des} to $r_\star$ is deformed using *Banach's Fixed Point Theorem* [12] in order to obtain $r^{Act} \equiv f(r_\star) = r^{Des}$.

Later the *Sigmoid Generated Fixed Point Transformation (SGFPT)* has been presented that can be used in nonlinear adaptive control of *Single Input – Single Output (SISO)* as well as *Multiple Input - Multiple Output (MIMO)* systems [13]. Also the applied sigmoid function have been reconsidered, which has crucial significance in the adaptive deformation [14].

The the excellent properties of soft computing methods, such as fuzzy logic, neural networks, evolutionary computing, provide a wide range of tools for addressing issues of establishing relationship between measurements, assigning error bars to predictions, integrating information from various sources with a varying degree of uncertainty, etc. [15]. Recently, most of the deterministic model buildings are increasingly replaced by soft-computing methods [16]. In spite of the classical hard-computing methods the intelligent methodologies are able to deal with imprecisions, uncertainties and partial truth by an efficient and robust way. Fuzzy logic is widely used for modeling complex and ill-defined systems. The core concept relies on the application of linguistic variables, see [17]. In this paper a possible combination of the fuzzy modeling and the SGFPT control strategy has been shown. The inverted pendulum system served as a nonlinear paradigm. The *"affine"*, and a *"soft computing-based"* model of the pendulum system were investigated. Simulation results have revealed that the proposed method fulfills the performance criteria.

II. Adaptive Controller using Fuzzy Modeling and the Sigmoid Generated Fixed Point Transformation

A. Principles of the Robust Fixed Point Transformation

This section shortly outlines the mathematical principles behind the *"Sigmoid Generated Fixed Point Transformation"* control method. At first, let us consider a monotonic increasing, bounded and smooth $g(x) : \mathbb{R} \mapsto \mathbb{R}$ "sigmoid" function. For some $K > 0$ and $D > 0$ consider a function $F(x) \overset{def}{=} g^{-1}(g(x) - K) + D$, in which the inverse function of $g()$ is denoted by $g^{-1}()$. The solution of $F(x) = x$ is the *fixed point* of $F(x)$. For the case of $g(x) \overset{def}{=} \tanh(x)$, $K = 0.5$, and $D = 0.6$ an example is displayed in Fig. 1
From Fig. 1 it can be observed, that this function has two fixed points, at the first one $\left|\frac{dF}{dx}\right| > 1$ while at the other one $\left|\frac{dF}{dx}\right| < 1$. A function is called *contractive* in the case of real, differentiable $\varphi : \mathbb{R} \mapsto \mathbb{R}$ functions if $\exists 0 \leq K < 1$, hence $\forall a, b \in \mathbb{R}$ $|g(a) - g(b)| < K|a - b|$. It is evident, if $\exists 0 \leq K < 1$ so that $\left|\frac{d\varphi}{dx}\right| < K$, thus the integral estimation below can be used:

$$|\varphi(b)-\varphi(a)| = \left|\int_a^b \frac{d\varphi}{dx} dx\right| \leq \int_a^b \left|\frac{d\varphi}{dx}\right| dx \leq K|b-a| \ . \quad (1)$$

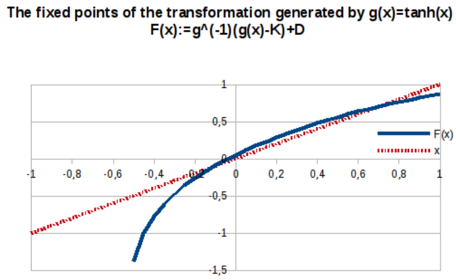

Fig. 1. The fixed points of $F(x)$

Obviously, $F(x)$ is contractive around the fixed point at about $x = 0.7$, so the sequence $\{x_0, x_1 \overset{def}{=} F(x_0), \ldots, x_{n+1} \overset{def}{=} F(x_n), \ldots\}$ is a Cauchy sequence since $\forall L \in \mathbb{N}$

$$\begin{aligned} &|x_{n+L} - x_n| = |F(x_{n-1+L}) - F(x_{n-1})| \leq \\ &\leq K\,|x_{n-1+L} - x_{n-1}| \leq \ldots K^n\,|x_L - x_0| \to 0 \\ &\text{as } n \to \infty \ . \end{aligned} \quad (2)$$

Since the real numbers forms a complete space there $\exists x_* \in \mathbb{R}$ so that $x_n \to x_*$. Based on these considerations, the following fixed point transformation is proposed using $F(x)$ for adaptive control in the case of slowly varying desired response r^{Des}:

$$r_{n+1} = G(r_n) \overset{def}{=} F\left(A\left[f(r_n) - r^{Des}\right] + x_*\right) + r_n - x_* \quad (3)$$

, where $A \in \mathbb{R}$ is a parameter. When $r_\star$ is the solution of the control task, i.e. $f(r_\star) - r^{Des} = 0$ then $G(r_\star) = r_\star$. Since $F(x_*) = x_*$, this fixed point of the function G is this solution. In order to ensure the convergence of the series $\{r_n\}$, function G must be contractive, i.e. the relation $\left|\frac{dG}{dr}\right| < 1$ must be guaranteed. This contractivity can be obtained by appropriately setting the value of parameter A:

$$\frac{dG}{dr} = A \frac{dF\left(A\left[f(r) - r^{Des}\right] + x_*\right)}{dx} \frac{df}{dr} + 1 \ . \quad (4)$$

This fixed point transformation has been extended to MIMO systems in [13], when $f, r \in \mathbb{R}^n$, $n \in \mathbb{N}$ (n is degree of freedom (DoF) of the controlled system). From this point on the iteration generates sequences as $\{r(i) \in \mathbb{R}^n | i = 0, 1, \ldots\}$ by using the direction of the *response error* in the i^{th} step of the iteration as

$$h(i) \overset{def}{=} f(r(i)) - r^{Des}, e(i) \overset{def}{=} \frac{\mathfrak{A}h(i)}{\|\mathfrak{A}h(i)\|}, \quad (5)$$

where $\|h\|$ is the Frobenius norm:

$$\begin{aligned} &r(i+1) = \tilde{G}(r(i)) \overset{def}{=} \\ &\overset{def}{=} \left[F(A\|\mathfrak{A}h(i)\| + x_*) - x_*\right] e(i) + r(i) \ . \end{aligned} \quad (6)$$

Then, the solution of the control task is the fixed point of $\tilde{G}(r)$ when $f(r_\star) - r^{Des} \equiv h(i) = 0$ then $r(i+1) = r(i) = r_\star$.

This condition can be valid for a lot of physical systems. In (5) and (6) $\mathfrak{A}$ is a diagonal matrix with positive main diagonals that can be tuned to improve the convergence properties of the controller. (In the original approach it was the unit matrix.) Its matrix elements can be tuned by observing little fluctuations in the convergence of the adaptive signal when these main diagonals are too big. These fluctuations are revealed as a negative content in a forgetting buffer as it was done in [18]. Furthermore, the stability conditions for MIMO systems were discussed in an earlier paper [13]. Because the former technique worked in bounded region of attraction around $r_\star$ that formally could not guarantee global stability. In order to address this question, the so-called Strectched Sigmoid Fuction has been investigated [14]. It was assumed that for the first element of the iteration x_0 there exists x_1 for which $g(x_0) - K = g(x - D)$. It was found to be valid for most of the x_0 values but not for each of them. This limitation could be evaded by introducing the following stretched function

$$F(x) = B\tanh(a(x+b)) + K \tag{7}$$

with $a, b > 0$. For allowing further improvements and a more precise positioning of the function in the vicinity of the solution of the control task, the next, new type of function has been introduced in [19]

$$F(x) = \mathrm{atanh}(\tanh(x+D)/2). \tag{8}$$

B. *The System under Consideration*

In this paper the inverted pendulum system serves as a benchmark problem. The generalized coordinates are $q_1\,[rad]$, $q_2\,[rad]$ and the generalized forces are torque signals: $Q_1\,[N \cdot m]$, $Q_2\,[N \cdot m]$. The equations of motion are given in (9).

$$\begin{pmatrix} mL^2 & 0 \\ 0 & mL^2\sin^2 q_1 \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} -mL^2 \sin q_1 \cos q_1 \dot{q}_2^2 + mgL\sin q_1 \\ 2mL^2 \sin q_1 \cos q_1 \dot{q}_1 \dot{q}_2 \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \tag{9a}$$

The system parameters are given in Table I.

Parameter	Exact Value	Approximate Value
mass m	$0.5\,kg$	$1\,kg$
lenght L	$1.5\,m$	$1\,m$
gravitational acceleration g	$9.81\,\frac{m}{s^2}$	$10\,\frac{m}{s^2}$

TABLE I
THE SYSTEM MODEL AND ITS PARAMETERS

C. *On the Adaptive Control Strategy*

The *constant control parameters* are given in Table II. The simulations were made by the use of the package *"Julia"* with a sequential code using Euler integration method with a fixed step length $10^{-4}\,s$. The constant control parameters are given in Table II.

Parameter	Value
Λ	$4\,s^{-1}$
D	0.3
δt time delay in learning	$10^{-3}\,s$

TABLE II
SETTING THE CONSTANT CONTROL PARAMETERS

A PID-type relaxation can be prescribed for the tracking error of q. Let $\mathbb{R} \ni \Lambda > 0$, and let

$$e(t) \stackrel{def}{=} q^N(t) - q(t) \ , \tag{10a}$$

$$e_{int}(t) = \int_{t_0}^{t} \left(q^N(\xi) - q(\xi)\right) d\xi \ , \tag{10b}$$

$$\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)^2 e(t) = 0 \Rightarrow \tag{10c}$$

$$\ddot{q}^{Des} = \ddot{q}^N + \Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) \ . \tag{10d}$$

The adaptive controller at first deforms $\ddot{q}^{Des}$ into $\ddot{q}^{Def}$, and uses the approximate dynamic model for the calculation of the control forces for this deformed value. According to (9), for this excitation the controlled system responses wit the *"Realized"* response $\ddot{q}$. On the sequel the function $\ddot{q} = f\left(\ddot{q}^{Def}\right)$ is referred to as the *"response function"* of the system under control. The adaptive deformation is based on the measurements of these responses. The controller used the function $F(x) = \mathrm{atanh}\left(\frac{\tanh(x+D)}{2}\right)$ for making the adaptive deformation.

III. Simulation Results

A. *Results for the affine model*

At first, we investigate the affine model. The simulation results are displayed in Figs. 2-4. The adaptive strategy has been compared with a non-adaptive one. It is evident that the modeling errors cause errors in the computed torque signals and corrupt the precision of trajectory tracking.

B. *Results for the soft computing-based model*

In this case the $\sin(x)$ and the $\cos(x)$ functions were approximated by fuzzy rules as $\mu_s(x)$ and $\mu_c(x)$ over a bounded region according to Fig. 6.
In the *approximate dynamic model* the additional terms were constants just as in the case of the "affine model", but the function $\sin(x)$ in the inertia matrix was approximated:

```
function ApprMod(q,q_p,q_ppDes)
  global ma
  global La
  global ga
  H=zeros(2,2)
```

```
  sq1=mus(q[1,1])
  H[1,1]=ma*La^2;
  H[2,2]=ma*La^2*sq1^2;
  h=[1.0;1.0];
  return H*q_ppDes+h;
end
```

C. Results for the fully soft computing-based model

In the *approximate dynamic model* in each term the functions $\sin(x)$ and $\cos(x)$ were approximated by the fuzzy approximations $\mu_s(x)$ and $\mu_c(x)$ as

```
 function ApprMod(q,q_p,q_ppDes)
  global ma
  global La
  global ga
  H=zeros(2,2)
  sq1=mus(q[1,1])
  cq1=muc(q[1,1])
  H[1,1]=ma*La^2;
  H[2,2]=ma*La^2*sq1^2;
  h=[1.0;1.0];
  h[1,1]=-ma*La^2*sq1*cq1*q_p[2,1]^2+
  +ma*ga*La*sq1;
  h[2,1]=2*ma*La^2*sq1*cq1*q_p[1,1]*q_p[2,1];
  return H*q_ppDes+h;
end
```

IV. Conclusions

In this paper the adaptive control design using fuzzy modeling and the *" Sigmoid Generated Fixed Point Transformation* based control approach has been preseneted. In the last years the *" Sigmoid Generated Fixed Point Transformation (SGFPT)"* has been introduced for replacing the Lyapunov technique. This systematic method has been proposed for the generation of whole families of Fixed Point Transformations. In addition it has been extended from Single Input Single Output (SISO) to Multiple Input Multiple Output (MIMO) systems. Recently, several model building issues are increasingly replaced by soft-computing based methods. In the present paper the *"affine"*, and a *"soft computing-based"* model of the inverted pendulum system under control has been investigated. In the soft computing-based model the $\sin(x)$ and the $\cos(x)$ functions were approximated by fuzzy rules. In the case of the *"fully soft computing-based"* model, in the *approximate dynamic model* in each term the functions $\sin(x)$ and $\cos(x)$ were approximated by the fuzzy approximations. Simulation results have revealed that the application of fuzzy approximation in the SGFPT-type control design is far promising. Additionally, the results also validate the applicability and efficiency of the proposed technique.

Acknowledgment

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References

[1] A. Lyapunov, *A general task about the stability of motion. (in Russian).* Ph.D. Thesis, University of Kazan, Tatarstan (Russia), 1892.

[2] H. Khalil, *Nonlinear Systems (2^{nd} Ed.).* Hall: Upper Saddle River, 1996.

[3] R. Kamnik, D. Matko, and T. Bajd, "Application of model reference adaptive control to industrial robot impedance control," *Journal of Intelligent and Robotic Systems*, vol. 22, pp. 153–163, 1998.

[4] J. Somló, B. Lantos, and P. Cát, *Advanced Robot Control.* Akadémiai Kiadó, Budapest, 2002.

[5] C. Nguyen, S. Antrazi, Z.-L. Zhou, and C. C. Jr., "Adaptive control of a stewart platform-based manipulator," *Journal of Robotic Systems*, vol. 10, no. 5, pp. 657–687, 1993.

[6] K. Hosseini-Suny, H. Momeni, and F. Janabi-Sharifi, "Model Reference Adaptive Control design for a teleoperation system with output prediction," *J Intell Robot Syst*, vol. DOI 10.1007/s10846-010-9400-4, pp. 1–21, 2010.

[7] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control.* Prentice Hall International, Inc., Englewood Cliffs, New Jersey, 1991.

[8] J. Tar, *Adaptive Control of Smooth Nonlinear Systems Based on Lucid Geometric Interpretation (DSc Dissertation).* Hungarian Academy of Sciences, Budapest, Hungary, 2012.

[9] ——, "Replacement of Lyapunov function by locally convergent robust fixed point transformations in model based control, a brief summary," *Journal of Advanced Computational Intelligence and Intelligent Informatics*, vol. 14, no. 2, pp. 224–236, 2010.

[10] J. Tar, J. Bitó, L. Nádai, and J. T. Machado, "Robust Fixed Point Transformations in adaptive control using local basin of attraction," *Acta Polytechnica Hungarica*, vol. 6, no. 1, pp. 21–37, 2009.

[11] J. Tar, J. Bitó, and I. Rudas, "Replacement of Lyapunov's direct method in model reference adaptive control with robust fixed point transformations," *In Proc. of the 14^{th} IEEE Intl. Conf. on Intelligent Engineering Systems, Las Palmas of Gran Canaria, Spain*, pp. 231–235, 2010.

[12] S. Banach, "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales (About the Operations in the Abstract Sets and Their Application to Integral Equations)," *Fund. Math.*, vol. 3, pp. 133–181, 1922.

[13] A. Dineva, J. Tar, A. Varkonyi-Koczy, and V. Piuri, "Generalization of a sigmoid generated fixed point transformation from siso to mimo systems," in *IEEE 19th International Conference on Intelligent Engineering Systems (INES2015), September 3-5 2015, Bratislava, Slovakia*, 2015, pp. 135–140.

[14] ——, "Adaptive control of underactuated mechanical systems using improved sigmoid generated fixed point transformation and scheduling strategy," in *IEEE 14th International Symposium on Applied Machine Intelligence and Informatics*, January 21-23, 2016, Herlany, Slovakia 2016, pp. 193–197.

[15] P. Wong, F. Aminzadeh, and M. Nikravesh, Eds., *Soft Computing for Reservoir Characterization and Modeling.* Springer- Verlag Berlin Heidelberg, 2002.

[16] H. Seki, "Nonlinear function approximation using fuzzy functional sirms inference model," *AIKED'12 Proceedings of the 11th WSEAS international conference on Artificial Intelligence, Knowledge Engineering and Data Bases*, pp. 201–206.

[17] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning," *Information Sciences*, vol. 8, no. 3, pp. 199–249.

[18] K. Kósi, J. Tar, and I. Rudas, "Improvement of the stability of RFPT-based adaptive controllers by observing "precursor oscillations"," *In Proc. of the 9^{th} IEEE Intl. Conf. on Computational Cybernetics, Tihany, Hungary*, pp. 267–272, 2013.

[19] A. Dineva, A. R. Varkonyi-Koczy, J. K. Tar, and V. Piuri, “Sigmoid generated fixed point transformation control scheme for stabilization of kapitzas pendulum system,” in *IEEE 20th Jubilee International Conference on Intelligent Engineering Systems: INES 2016, 2016.06.30-07.02., Budapest, Hungary*, 2016.

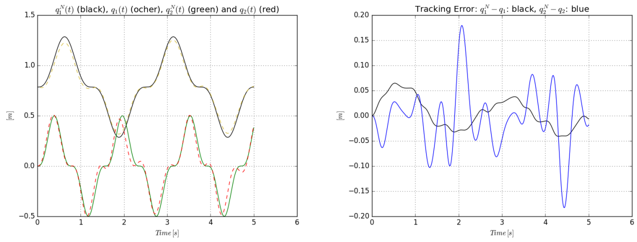

Fig. 2. Trajectory tracking of the non-adaptive controller for the *“affine model”*

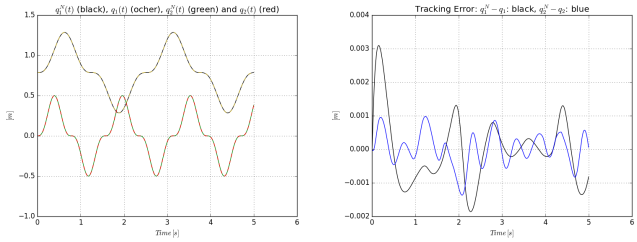

Fig. 3. Trajectory tracking of the adaptive controller for the *“affine model”*

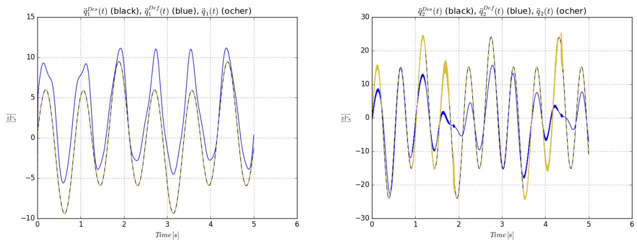

Fig. 4. The $\ddot{q}$ values of the adaptive controller for the *“affine model”*

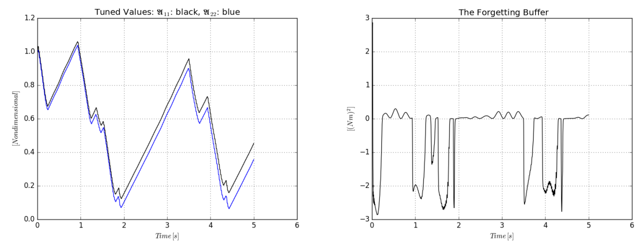

Fig. 5. The tuned parameters of the adaptive controller and the content of the forgetting buffer for the *“affine model”*

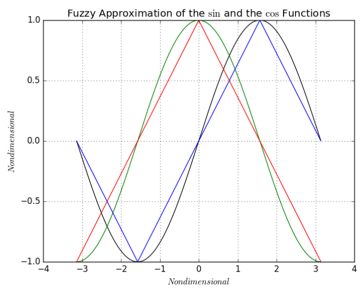

Fig. 6. Functions $\sin(x)$ (black), $\mu_s(x)$ (blue), $\cos(x)$ (green), and $\mu_c(x)$ (red)

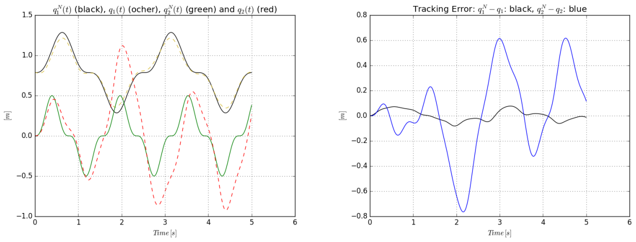

Fig. 7. Trajectory tracking of the non-adaptive controller for the *"soft computing-based model"*

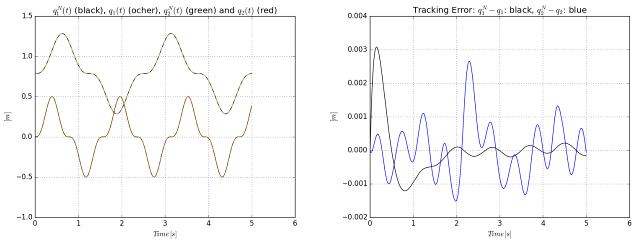

Fig. 8. Trajectory tracking of the adaptive controller for the *"soft computing-based model"*

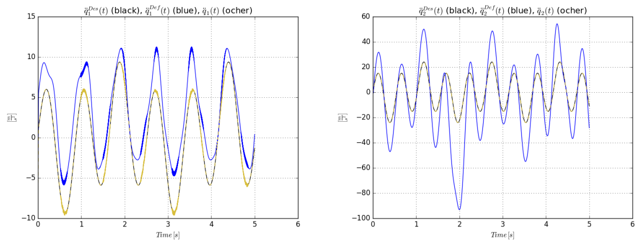

Fig. 9. The $\ddot{q}$ values of the adaptive controller for the *"soft computing-based model"*

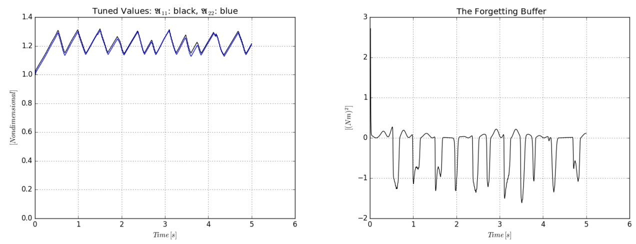

Fig. 10. The tuned parameters of the adaptive controller and the content of the forgetting buffer for the *"soft computing-based model"*

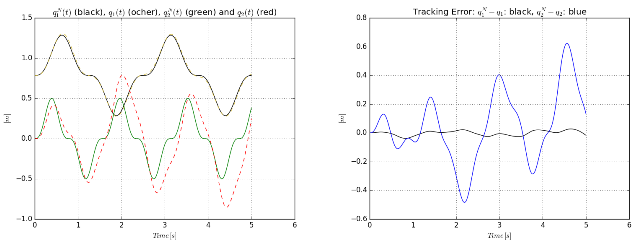

Fig. 11. Trajectory tracking of the non-adaptive controller for the *"soft computing-based model"*

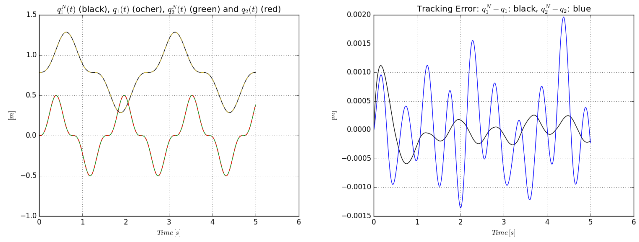

Fig. 12. Trajectory tracking of the adaptive controller for the *"fully soft computing-based model"*

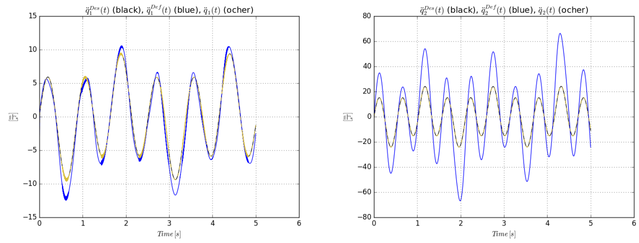

Fig. 13. The $\ddot{q}$ values of the adaptive controller for the *"fully soft computing-based model"*

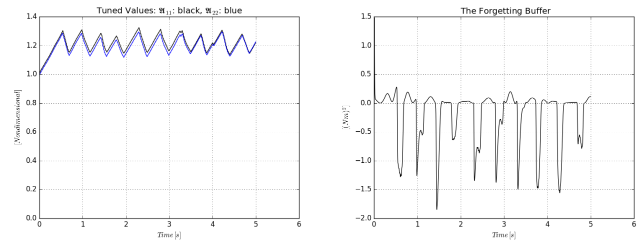

Fig. 14. The tuned parameters of the adaptive controller and the content of the forgetting buffer for the *"fully soft computing-based model"*