Test Chirp Signal Generation Using Spectral Warping

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Abstract

A DSP technique that transforms a digital signal by warping the frequency axis is discussed. The technique corresponds to a mapping of the samples in the z-*domain such that they are unevenly spaced around the unit circle. A spectral warping network has the property that a time-shifted impulse function is transformed into a chirp function. This can have useful applications in the analysis of the device-under-test frequency response in electronic testing of analogue and mixed-signal circuits/systems.*

1. Introduction

In simple words, spectral warping (SW) is a digital signal processing transform that shifts a signal's frequency components along the frequency axis. That is, it maps one frequency axis onto another. A. Oppenheim *et al* first proposed the transform in 1971 [1] and discussed its use for unequal-resolution spectral analysis. That work was expanded upon in [2, 3] and in a number of other papers, and more recently in [4, 5]. The transform has also been proposed as an analysis tool for mixed-signal testing [6].

This paper discusses the properties of SW and shows that a chirp signal is generated when an impulse undergoes SW. It then examines how such a chirp signal can be employed in analogue and mixed-signal testing to find a device-under-test's frequency response function.

2. A SW network

A SW transform can be realised by cascading all-pass filters, as shown in Figure 1. The time-reversed input sequence is fed through the network. The last sample of the sequence is taken at the output of each filter. These samples form the output sequence. Consequentially, the length of the output sequence is equal to the number of filters used. All the filters are identical, except for the first two, which have different zeroes.

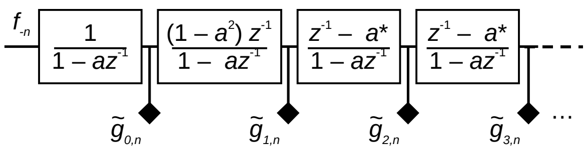

Figure 1. Network of all-pass filters

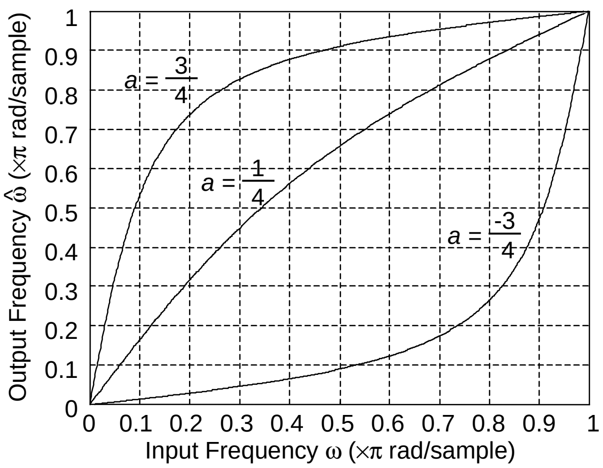

Figure 2. Frequency mapping

3. Properties of the SW transform

3.1. Frequency shifting

The frequency components of a signal that has undergone SW are shifted in the frequency domain. This distortion of the frequency axis is given by expression (1). The extent of the distortion is determined by the parameter *a*, which is called the warping factor.

$$\hat{\omega} = \theta(\omega) = \tan^{-1}\left[\frac{(1-a^2)\sin\omega}{(1+a^2)\cos\omega - 2a}\right] \quad -1 < a < 1 \qquad (1)$$

Distortion curves for some warping factors are given in Figure 2. It can be seen that the closer the magnitude of a is to unity, the more severe is the distortion. The symmetry between the curves for a particular warping factor and the negative of that warping factor is also apparent. This shows that the transform is reversible. An output signal produced by warping with a warping factor of a can be transformed back to the original by warping with $\tilde{a} = -a$.

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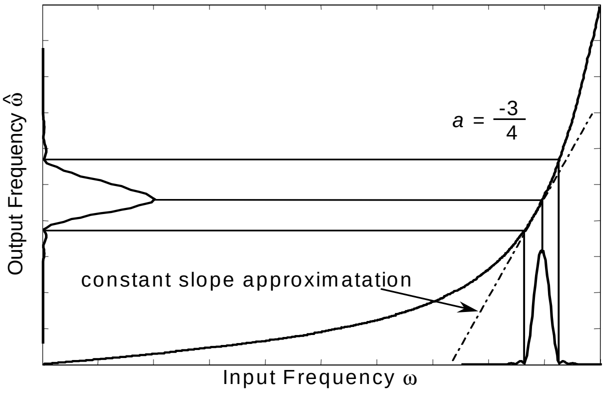

Figure 3. Spectrum distortion

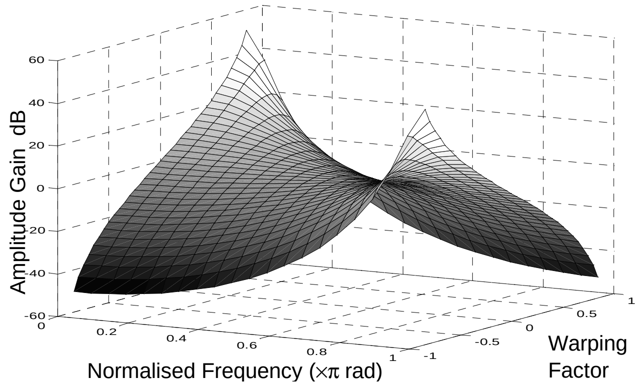

Figure 4. Magnitude response of SW transform

3.2. Time shifting and distortion of signal length

In order to process a segment of a continuous signal, it is multiplied by a windowing function in time domain. This corresponds to, in the frequency domain, convolution between the spectrum of the signal and that of the window. If the bandwidth of the signal is much narrower than the bandwidth of the windowing function, then the resulting spectrum takes on the shape of the spectrum of the windowing function, but remains at the same centre frequency as the original signal.

When the windowed signal is transformed using SW this spectrum is warped according to the curve (1). If the bandwidth of the windowing function is sufficiently limited, the SW frequency distortion curve can be well approximated by a constant slope function (as illustrated in Figure 3).

There is a reciprocal relationship between the time and frequency domains. This means that the duration of the signal is reduced if the bandwidth is increased, and visa versa. Therefore the function of the duration distortion is the reciprocal of the derivative of (1) and is presented by the following expression:

$$\psi(\omega) = (1 - 2a\cos\omega + a^2)/(1 - a^2) \tag{2}$$

A sinusoidal input signal of frequency ω that starts at time T_1 and finishes at T_2 will produce a sinusoidal output of frequency $\hat{\omega} = \theta(\omega)$, spanning the time from $\hat{T}_1$ to $\hat{T}_2$. These values can be obtained by substituting T with T_1 and T_2 in the expression:

$$\hat{T}(T,\omega) = \frac{1 - 2a\cos\omega + a^2}{1 - a^2} \times T = \psi T \tag{3}$$

3.3. Frequency Response

The energy of each frequency component of an original signal is preserved during SW transformation (providing there are enough output samples to ensure that the signal is not truncated). Therefore when a frequency component is stretched out in time it must be attenuated. Similarly, it is amplified if it is squashed up in time. The expression for the magnitude gain response (shown in Figure 4) is the reciprocal of (2), hence the derivative of (1):

$$|H(\omega)| = (1 - a^2)/(1 - 2a\cos\omega + a^2) \tag{4}$$

This result was first stated in [3], along with the phase response (M is the number of filters):

$$\Phi(\omega) = (1 - M)\tan^{-1}\left[\frac{(1 - a^2)\sin\omega}{(1 + a^2)\cos\omega - 2a}\right] \tag{5}$$

4. Chirp signal generation

An impulse at time T_0 can be constructed with a linear combination of cosine functions. The addition of these functions produces δ value at $t = T_0$ and gives 0 (from the functions cancelling each other) for $t \neq T_0$. The discrete-time impulse, or unit impulse (presented by the Dirac delta function), can be expressed as:

$$\delta[n - T_0] = \frac{2}{N}\sum_{i=1}^{N/2}\cos\left(\frac{2\pi i}{N}(n - T_0)\right) \quad n = 0,1,\ldots,N-1 \tag{6}$$

Because superposition holds for SW, the transformation of an impulse can be analysed by observing the warping of the sinusoids that make it up. It is useful to consider the sinusoidal components as spanning the whole sequence, i.e., from $n = T_1 = 0$ to $n = T_2 = N - 1$. Each transformed component still begins at the origin ($\hat{T}_1 = \psi \times 0 = 0$), but lasts until $\hat{T}_2 = \psi T_2$. The phase of each component at the origin is also unchanged.

The sum of the sinusoids is no longer zero at all but one of the points. Although, components of similar frequency do, to some extent, constructively interfere at a certain time and destructively interfere at other times. If two components of frequencies ω_a and ω_b are transformed to components of frequencies $\hat{\omega}_a$ and $\hat{\omega}_b$ respectively, the point where they originally coincided, T_0, is transformed to $\hat{T}_{0a}$ and $\hat{T}_{0b}$ respectively. If $\omega_a \approx \omega_b$, then

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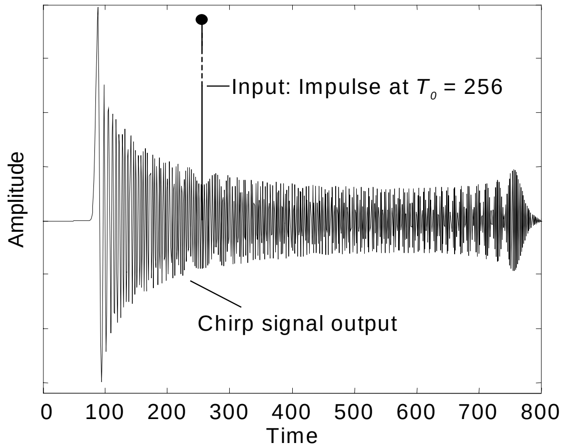

Figure 5. Chirp signal generated by warping an impulse with a warping factor of *a* = 0.5

$\hat{T}_{0a} \approx \hat{T}_{0b}$ and constructive interference will occur at that time.

Thus if an impulse is transformed, the time in the output sequence at which each frequency will have its maximum power is given by (3), substituting T_0 for T.

The result is a signal in which the low frequencies have a maximum at one end and the high frequencies have a maximum at the other. The time at which each frequency has its maximum can be found by replacing ω with its expression in terms of the distorted frequency.

$$\hat{t}(\hat{\omega}) = \frac{1-2a\cos\left(\tan^{-1}\left[\frac{(1-a^2)\sin\hat{\omega}}{(1+a^2)\cos\hat{\omega}+2a}\right]\right)+a^2}{1-a^2}T_0 \quad (7)$$

Figure 5 shows a chirp signal generated by SW. The warping factor is positive, so the chirp begins with low frequencies and finishes with high frequencies. The frequency distribution is shown in Figure 6.

It can be seen from Figure 5 that beating appears towards the tail of the chirp signal. By this time only the high-frequency components are present. The few remaining frequency components move in and out of phase, causing constructive and then destructive interference that results in the beating.

Up to a certain point all the frequency components are present in the output signal and they cancel each other out. The chirp signal begins when the first frequency component finishes. This component will be the one relating to either $\omega = \omega_{min}$ or $\omega = \pi\omega_{min}$, depending on the sign of a. ω_{min} is the frequency resolution, given below.

$$\omega_{min} = \left(\frac{N}{2}-1\right)\Big/\left(\frac{N}{2}\right) \quad (8)$$

We can assume ω_{min} is close to 1 and substitute $\omega = 0$ or π, as appropriate, into (3) to find the start of the chirp, T_1, as a function of the time of the impulse.

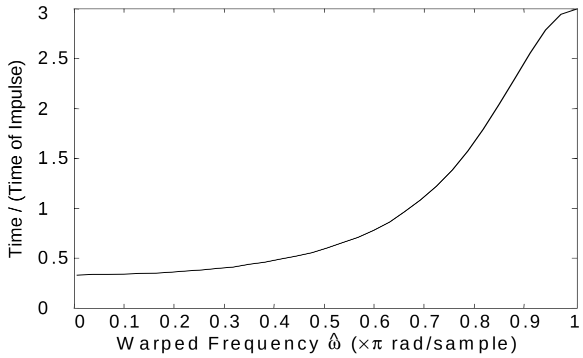

Figure 6. Locations of frequencies in chirp relative to T_0 (*a* = 0.5)

$$T_1 = \frac{(1-|a|)^2}{1-a^2} \times T_0 \quad (9)$$

The end of the chirp can be found in a similar manner:

$$T_2 = \frac{(1+|a|)^2}{1-a^2} \times T_0 \quad (10)$$

The duration of the chirp is $\Delta T = T_2 - T_1$. To produce a chirp of a given length, with a given warping factor a, the time the impulse must occur is $T_0 = (1-a^2)/(4a) \times \Delta T$.

5. Application of SW chirps in analogue and mixed-signal testing

The frequency response of an electronic analogue system can be found by applying an impulse to the input and measuring the output response. In order to provide good accuracy of the measurement, the generated impulse has to be of substantial energy. Because all the energy in an impulse signal is concentrated in one point in time, this implies a very high magnitude of the impulse (ideally it has to be of unlimitedly high magnitude as the Dirac delta function). This means that applying an impulse to the device-under-test (DUT) can be damaging to the device. The use of a SW generated chirp signal eliminates this problem. The signal is essentially equivalent to the unit impulse; thus it contains frequencies from the entire spectrum, while in contrast to the ideal pulse its amplitude is well limited.

To find a system's frequency response, a test chirp is generated by spectrally warping an impulse, using a warping factor a. This signal is then passed through a DUT. This gives the system's 'chirp response'. The Fourier transform of the chirp response has the same magnitude as the system's frequency response. This is because each frequency component of the chirp has equal energy. The phase information is altered due to the time shifting.

To recover the phase data the response is warped with $-a$. This produces a warped impulse response that is time-

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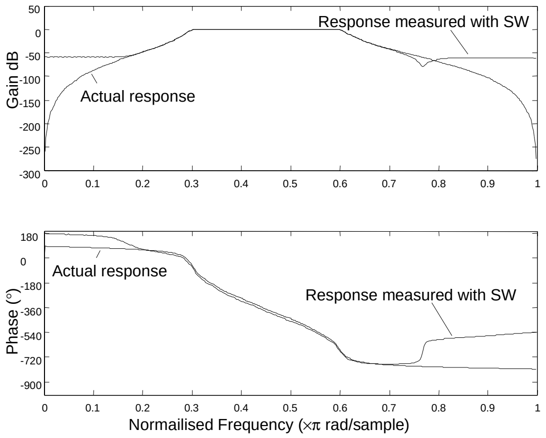

Figure 7. The frequency response of a system: known response and response measured using SW (*a* = 0.5)

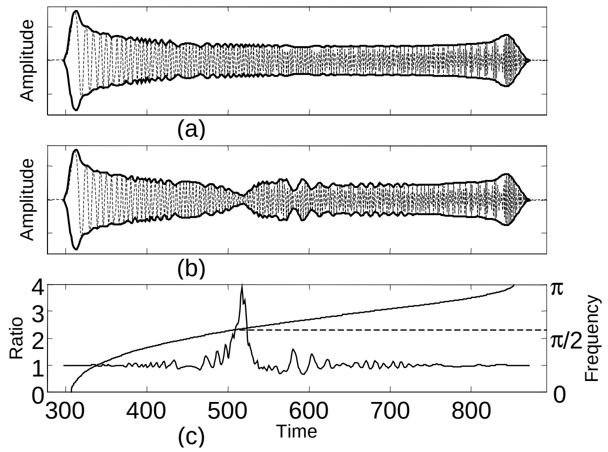

Figure 8. An envelope of a chirp signal (a) and a chirp response (b) for a band-stop filter, and the ratio of the envelopes overlaid with the frequency versus time curve (c)

shifted by T_0. Shifting this to start at the origin and then warping it (using the original warping factor a) will make it identical to an impulse response found in the conventional manner. As usual, the system's frequency response can be found by studying the Fourier transform of the impulse response.

As an example, the Fourier transform of the impulse response of an arbitrary system was found using SW, as described above. The results are plotted in Figure 7, together with the known frequency response of the system.

It is possible to estimate a system's magnitude response from its chirp response without calculating a Fourier transform. Because the time at which each frequency is dominant in the chirp is known from equation (7), an indication of the magnitude response can be obtained by comparing the envelope of the chirp response with that of the chirp signal. Figure 8 gives an example of a chirp response of a band-stop filter with a very narrow stop-band. Only one section of the chirp response's envelope differs significantly from the envelope of the original chirp. This occurs at the time when the stop-band frequency is dominant in the chirp. This frequency can be found by overlaying the frequency versus time curve for the particular chirp. (The known stop band for this example filter is 0.57π to 0.58π). This approximation is only valid if the phase shift introduced by the system is not excessive.

6. Conclusion

In this paper a review of the Spectral Warping digital signal processing transform is given. The transform shifts the frequency components of a signal along the frequency axis, while maintaining the general shape of the spectrum. The duration and amplitude of each frequency component are correspondingly affected.

The SW transform can be used to find the impulse response of a system, without the risk of overloading and damaging the system, as might happen when applying an impulse. A suitable chirp signal is generated and applied to the system. The output is fed through two SW networks to produce the impulse response. If only the magnitude response is required then no SW is needed – the Fourier transform of the chirp response produces the true magnitude response. An estimate of a system's magnitude response can be found from the envelope of the output.

The proposed generation/analysis technique can be employed in electronic testing of analogue and mixed-signal circuits and systems.

7. References

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