Non-conventional Control Design by Sigmoid Generated Fixed Point Transformation Using Fuzzy Approximation

Adrienn Dineva, József K. Tar, Annamária Várkonyi-Kóczy, János T. Tóth and Vincenzo Piuri

Abstract Lyapunov's 2nd or Direct method is recognized as being the primary tool of adaptive control of nonlinear dynamic systems. The great majority of the adaptive nonlinear control design rest on Lyapunov's stability theorem. Recent findings have revealed that the Robust Fixed Point Transformation-based method can succesfully replace the Lyapunov technique. Later the "*Sigmoid Generated Fixed Point Transformation (SGFPT)*" has been introduced. This systematic method has been proposed for the generation of whole families of Fixed Point Transformations. Its extension from Single Input Single Output (SISO) to Multiple Input Multiple Output (MIMO) systems has also been given. In recent times, the great majority of model building issues are replaced by "*Soft Computing*" techniques. In contrast to the classical mathematical methods the intelligent methodologies are able to cope with ill-defined systems, disturbances and missing information by an efficient and robust way. Especially fuzzy logic has become to be used to model complex systems. This

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contribution makes an attempt to utilize the advantages of fuzzy approximation in the SGFPT control design. The theoretical investigations are validated by the adaptive control of the inverted pendulum. Comparative analysis have been carried out between the "*affine*" and the "*soft computing-based*" models. Results of numerical simulations confirm the applicability and efficiency of the proposed method.

Keywords Sigmoid Generated Fixed Point Transformation · Fuzzy modeling Nonlinear control · Iterative learning · Fixed point transformation

1 Introduction

Lyapunov's *direct* method (or commonly referred to as the *second method* of Lyapunov) is a widely used technique in the analysis of the stability of the motion of the non-autonomous dynamic systems of equation of motion as $= f(x, t)$. Its main advantage that it does not require to analytically solve the equations of motion [1, 2]. Instead of the analytical solution, in most of the practical applications the uniformly continuous nature and non-positive time-derivative of a positive definite Lyapunov-function V is constructed of the tracking errors and modeling errors of the systems parameters that are assumed in the $t \in [0, \infty)$ domain. From this point the convergence $\dot{V} \to 0$ can be concluded according to Barbalat's lemma. However, in spite of its advantages, the application of Lyapunov's second method is far difficult and requires great practice for finding the appropriate Lyapunov function candidate. Additionally, the automization of such algorithm is challenging and also the computational need of realizing the method is significant.

Though, most of the control design techniques are based on Lyapunov's theorem. For instance, the *Model Reference Adaptive Controller (MRAC)* applies it for tuning the feedback parameters [3–6]. The classical methods, such as the *Slotine-Li Adaptive Robot Controller (SLARC)* and also the *Adaptive Inverse Dynamics Robot Controller (AIDRC)* control strategies apply Lyapunov's second method for tuning the parameters of the analytical system model [7]. An exhaustive study on the application of Lyapunov's direct method discussing the relating practical issues can be found in [8]. Originally, the Robust Fixed Point Transformation (RFPT) based control method has been introduced for replacing Lyapunov's second method [9–11]. The base of the theory of the Fixed Point Transformation-based control approach relies on the Expected-Realized Response Scheme. Some control tasks can be formulated by using the concept of the appropriate excitation of the controlled system to which it is expected to respond with some desired response r^d. The appropriate excitation can be computed by using the inverse dynamic model of the system. Since normally this inverse model is neither complete nor exact, the actual response determined by the systems dynamics, results in a realized response r^r that differs from the desired one. The idea of the RFP-based iterative adaptive learning approach is that the controller normally can manipulate or deform the input value from r^d to $r^d_\star$, so that $r^d(r^d_\star)$. This situation can be maintained by using some local deformation that has been realized by fixed point transformation [8]. The other specific feature of the RFPT-based control method, that it strictly separates the "kinematic" and "dynamic" aspects of the control task and predefines the "desired system response" by the use of purely kinematic terms.

The necessary dynamic terms are estimated using the approximate dynamic model of the system under control. The realization of the kinematic specification then based on these dynamic terms. The undesired effects of modeling imprecisions and unknown external disturbances are compensated by the above mentioned adaptive deformation of the *desired response* r^{Des} as the input of the approximate model. By this deformation, the desired response can be obtained as the realized one. The adaptive deformation is carried out by some kinematically expressed trajectory error reduction, i.e. *desired response* r^{Des} and the *actually observed response* r^{Act} are compared. Then, the *actually observed response* is deformed due to the exact system under control, formally $r^{Act} = f(r^{Des}, \dots)$ where the symbol "…" stands for the unknown state variables, for e.g. coupled subsystems, disturbances, etc. Due to the additive noises, etc., $r^{Act} \neq r^{Des}$. Hence, the input of the response function from r^{Des} to $r_\star$ is deformed using *Banach's Fixed Point Theorem* [12] in order to obtain $r^{Act} \equiv f(r_\star) = r^{Des}$.

Following, the *Sigmoid Generated Fixed Point Transformation (SGFPT)* has been introduced for adaptive control of nonlinear *Single Input–Single Output (SISO)* and *Multiple Input–Multiple Output (MIMO)* systems [13], which allows generating a whole family of fixed point transformations. Besides, a new function has been presented for the adaptive deformation, that gives significant enhancement of the performance [14].

The great popularity of soft computing methods, such as fuzzy logic, neural networks, genetic algorithms, is due to their simplicity and versatility; these methods have the ability of modeling and analyzing ill-defined problems by a cost-effective way [15]. In the past it has been proved that most of the deterministic model buildings can be effectively replaced by soft-computing methods [16], especially with fuzzy models [17]. Fuzzy modeling is of primary importance in fuzzy theory and has many interpretations [18–20]. For instance, the widely known Takagi-Sugeno models are great universal approximators for some nonlinear behaviour [21]. By the use of Lyapunov stability theorem, a large number of papers have been published on the synthesis of fuzzy modeling and control [22]. In comparison with the classical hard-computing methods the intelligent methodologies are able to deal with imprecisions and uncertainties.

In this short contribution we demonstrate the possible cooperation of fuzzy modeling and the SGFPT control design. Investigations focus on both the "*affine*", the "*soft computing-based*" and the "*fully soft computing-based*" models of the inverted pendulum. Finally, numerical simulations have been shown for validating the usability of the proposed method.

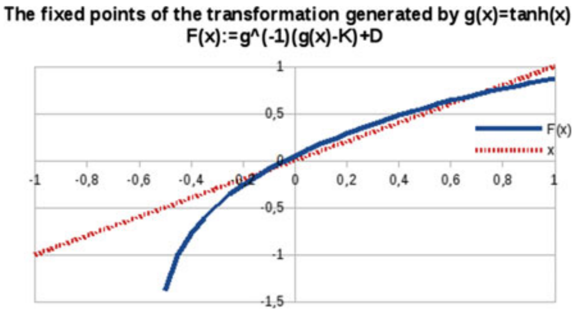


Fig. 1 The fixed points of $F(x)$

2 Combination of Fuzzy Modeling and Sigmoid Generated Fixed Point Transformation

2.1 The Concept of Sigmoid Generated Fixed Point Transformation

This section gives a brief summary of the mathematical background of the "*Sigmoid Generated Fixed Point Transformation*" control method. At first, let us consider a monotonic increasing, bounded and smooth $g(x) : \mathbb{R} \mapsto \mathbb{R}$ "sigmoid" function. For some $K > 0$ and $D > 0$ consider a function $F(x) \overset{def}{=} g^{-1}\left(g(x) - K\right) + D$, in which the inverse function of $g()$ is denoted by $g^{-1}()$. The solution of $F(x) = x$ is the *fixed point* of $F(x)$. For the case of $g(x) \overset{def}{=} \tanh(x)$, $K = 0.5$, and $D = 0.6$ an example is displayed in Fig. 1

It is clear from Fig. 1, that this function has two fixed points. The first one at $\left|\frac{\mathrm{d}F}{\mathrm{d}x}\right| > 1$ and the second one at $\left|\frac{\mathrm{d}F}{\mathrm{d}x}\right| < 1$. A function is called *contractive* in the case of real, differentiable $\varphi : \mathbb{R} \mapsto \mathbb{R}$ functions if $\exists 0 \leq K < 1$, hence $\forall a, b \in \mathbb{R}$ $|g(a) - g(b)| < K|a - b|$. It is evident, if $\exists 0 \leq K < 1$ so that $\left|\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right| < K$, therefore the following integral estimation can be applied:

$$|\varphi(b) - \varphi(a)| = \left|\int_a^b \frac{\mathrm{d}\varphi}{\mathrm{d}x}\mathrm{d}x\right| \leq \int_a^b \left|\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right| \mathrm{d}x \leq K|b - a| \quad . \tag{1}$$

Clearly, $F(x)$ is contractive around the fixed point at about $x = 0.7$, hence the sequence $\{x_0, x_1 \overset{def}{=} F(x_0), \ldots, x_{n+1} \overset{def}{=} F(x_n), \ldots\}$ is a Cauchy sequence since $\forall L \in \mathbb{N}$

$$\begin{gathered} \left|x_{n+L} - x_n\right| = \left|F(x_{n-1+L}) - F(x_{n-1})\right| \leq \\ \leq K\left|x_{n-1+L} - x_{n-1}\right| \leq \ldots K^n \left|x_L - x_0\right| \to 0 \\ \text{as } n \to \infty \quad . \end{gathered} \tag{2}$$

Since the real numbers forms a complete space there $\exists x_* \in \mathbb{R}$ so that $x_n \to x_*$. Based on these considerations, the following fixed point transformation is proposed using $F(x)$ for adaptive control in the case of slowly varying desired response r^{Des}:

$$r_{n+1} = G(r_n) \stackrel{def}{=} F\left(A\left[f(r_n) - r^{Des}\right] + x_*\right) + r_n - x_*, \tag{3}$$

where $A \in \mathbb{R}$ is a parameter. When $r_\star$ is the solution of the control task, i.e. $f(r_\star) - r^{Des} = 0$ then $G(r_\star) = r_\star$. Since $F(x_*) = x_*$, this fixed point of the function G is this solution. In order to ensure the convergence of the series $\{r_n\}$, function G must be contractive, i.e. the relation $\left|\frac{\mathrm{d}G}{\mathrm{d}r}\right| < 1$ must be guaranteed. It has been shown, that this contractivity can be achieved by properly setting the value of parameter A:

$$\frac{\mathrm{d}G}{\mathrm{d}r} = A\frac{\mathrm{d}F\left(A\left[f(r) - r^{Des}\right] + x_*\right)}{\mathrm{d}x}\frac{\mathrm{d}f}{\mathrm{d}r} + 1 \ . \tag{4}$$

A simple possibility for applying the same idea of adaptivity systems for Multiple Input–Multiple output systems has been presented in [13], when $f,\ r \in \mathbb{R}^n$, $n \in \mathbb{N}$ (in which n denotes the degree of freedom (DoF) of the controlled system). In this case the iteration generates sequences as $\{r(i) \in \mathbb{R}^n | i = 0, 1, \dots\}$ by applying a sigmoid function projected to the direction of the *response error* in the ith control cycle as:

$$h(i) \stackrel{def}{=} f(r(i)) - r^{Des}, e(i) \stackrel{def}{=} \frac{\mathfrak{A}h(i)}{\|\mathfrak{A}h(i)\|}, \tag{5}$$

in which the $\|h\|$ is the norm (in Frobenius sense):

$$\begin{aligned} r(i+1) = \tilde{G}(r(i)) &\stackrel{def}{=} \\ \stackrel{def}{=} \left[F(A\|\mathfrak{A}h(i)\| + x_*) - x_*\right] e(i) + r(i) \ . \end{aligned} \tag{6}$$

The solution of the control task is the fixed point of $\tilde{G}(r)$ when $f(r_\star) - r^{Des} \equiv h(i) = 0$ then $r(i+1) = r(i) = r_\star$.

The convergence properties of the controller can be enhanced by appropriately tuning the diagonal matrix with positive main diagonals $\mathfrak{A}$ in Eqs. 5 and 6. In this soft-computing based method it has been modified. In the original SGFPT it was the unit matrix [13]. Its matrix elements can be tuned by observing little fluctuations in the convergence of the adaptive signal when these main diagonals are too big. These fluctuations can be recognized as a negative content in a forgetting buffer as it was done in [23]. Furthermore, the stability conditions for MIMO systems were discussed in an earlier paper [13]. Because the former technique worked in bounded region of attraction around $r_\star$ that formally could not guarantee global stability. In order to address this question, the so-called Strectched Sigmoid Function has been investigated [14]. It was assumed that for the first element of the iteration x_0 there exists x_1 for which $g(x_0) - K = g(x - D)$. It was found to be valid for most of the x_0

values but not for each of them. This limitation could be evaded by introducing the following stretched function in [24]

$$F(x) = B\tanh(a(x+b)) + K \tag{7}$$

with $a, b > 0$. For allowing further improvements and a more precise positioning of the function in the vicinity of the solution of the control task, the next, new type of function has been introduced in [24]

$$F(x) = \mathrm{atanh}(\tanh(x+D)/2). \tag{8}$$

2.2 Application Example: Adaptive Control of the Inverted Pendulum

For the numerical simulation the inverted pendulum system is under consideration. The inverted pendulum is a well-defined benchmark for nonlinear control and many variants of this problem is documented. The generalized coordinates are q_1 $[rad]$, q_2 $[rad]$ and the generalized forces are torque signals: Q_1 $[N \cdot m]$, Q_2 $[N \cdot m]$. The equations of motion are given in (9).

$$\begin{pmatrix} mL^2 & 0 \\ 0 & mL^2 \sin^2 q_1 \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} -mL^2 \sin q_1 \cos q_1 \dot{q}_2^2 + mgL \sin q_1 \\ 2mL^2 \sin q_1 \cos q_1 \dot{q}_1 \dot{q}_2 \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \tag{9a}$$

The system parameters are given in Table 1.

2.3 Realization of the Proposed Method

The simulations have been carried out by the use of the package "*Julia*". This dynamic language allows a very fast evaluation. For the calculations the Euler integration method have been used with a fixed step length 10^{-4} s. The constant control parameters are collected in Table 2.

Table 1 The system model and its parameters

Parameter	Exact value	Approximate value
Mass m (kg)	0.5	1.2
Length L (m)	1.6	1.7
Gravitational acceleration g ($\frac{\mathrm{m}}{\mathrm{s}^2}$)	9.81	10

Table 2 The constant control parameters

Parameter	Value
Λ	$4\,\mathrm{s}^{-1}$
D	0.3
δt time delay in learning	10^{-3} s

The PID-type relaxation can be defined for the tracking error of q. Let $\mathbb{R} \ni \Lambda > 0$, and let

$$e(t) \stackrel{def}{=} q^N(t) - q(t) \ , \tag{10a}$$

$$e_{int}(t) = \int_{t_0}^{t} \left(q^N(\xi) - q(\xi)\right) d\xi \ , \tag{10b}$$

$$\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)^2 e(t) = 0 \Rightarrow \tag{10c}$$

$$\ddot{q}^{Des} = \ddot{q}^N + \Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) \ . \tag{10d}$$

Firstly, the adaptive controller deforms $\ddot{q}^{Des}$ into $\ddot{q}^{Def}$. After, it applies the approximate dynamic model for the calculation of the control forces for this deformed value. According to (9), for this excitation the controlled system responses with the "*Realized*" response $\ddot{q}$. The function $\ddot{q} = f\left(\ddot{q}^{Def}\right)$ is called as the "*response function*" of the system under control. The adaptive deformation relies on the measurements of these responses. For the adaptive deformation the $F(x) = \operatorname{atanh}\left(\frac{\tanh(x+D)}{2}\right)$ function have been used.

3 Simulation Investigations

3.1 Performance Using the Affine Model

At first, we investigate the affine model. The trajectory tracking and trajectory tracking error are depicted in Figs. 2, 3 and 4. The adaptive strategy has been shown in comparison with a non-adaptive one. It can be seen, that the imprecisions of trajectory tracking are due to the errors of the computed torque signals that caused by the modeling errors (Fig. 5).

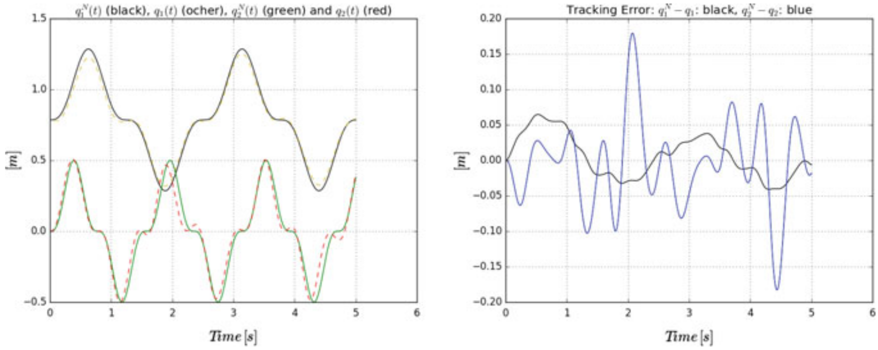


Fig. 2 Trajectory tracking of the non-adaptive controller for the "*affine model*"

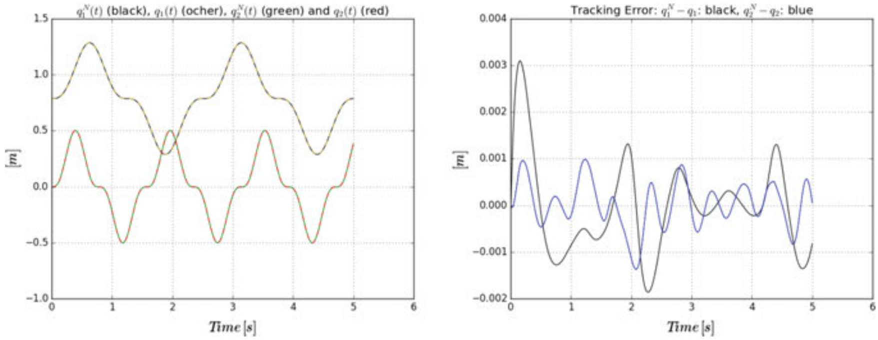


Fig. 3 Trajectory tracking of the adaptive controller for the "*affine model*"

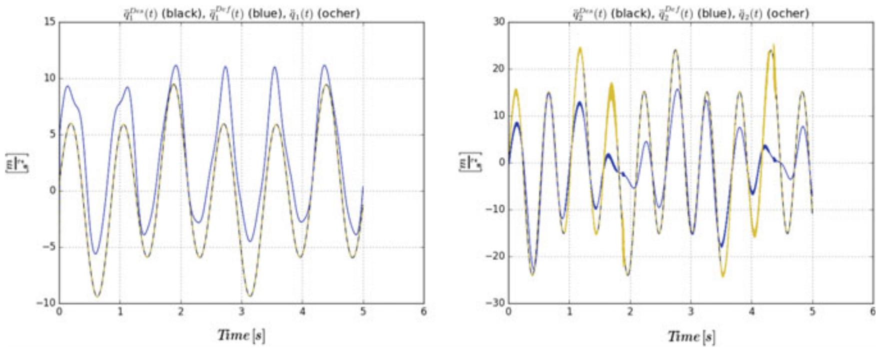


Fig. 4 The $\ddot{q}$ values of the adaptive controller for the "*affine model*"

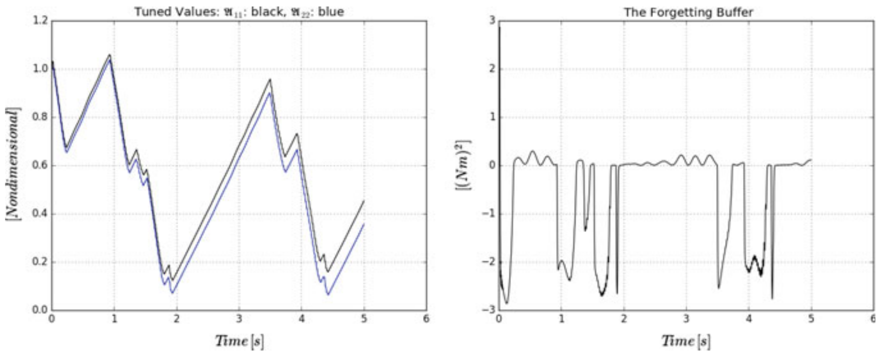
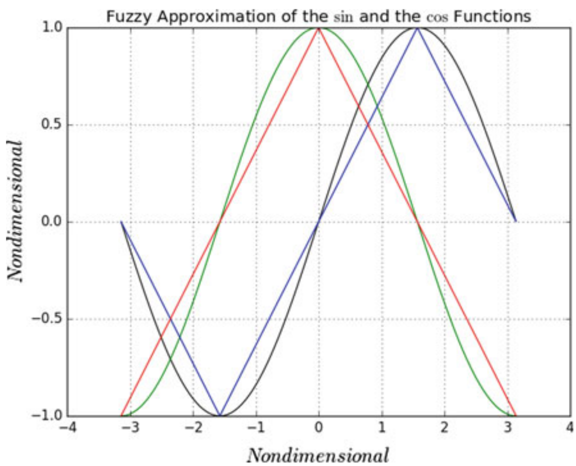


Fig. 5 The tuned parameters of the adaptive controller and the content of the forgetting buffer for the "*affine model*"

Fig. 6 Functions $\sin(x)$ (black), $\mu_s(x)$ (blue), $\cos(x)$ (green), and $\mu_c(x)$ (red)



3.2 Performance Using the Soft Computing-Based Model

In order to avoid the above mentioned errors the $\sin(x)$ and the $\cos(x)$ functions were approximated by fuzzy rules as $\mu_s(x)$ and $\mu_c(x)$ over a bounded region according to Fig. 6.

In the *approximate dynamic model* the additional terms remained constants just as in the case of the "affine model". In the inertia matrix the function $\sin(x)$ was replaced by fuzzy modeling as follows (Figs. 7 and 8):

```
function ApprMod(q,q_p,q_ppDes)
   global ma
   global La
   global ga
```

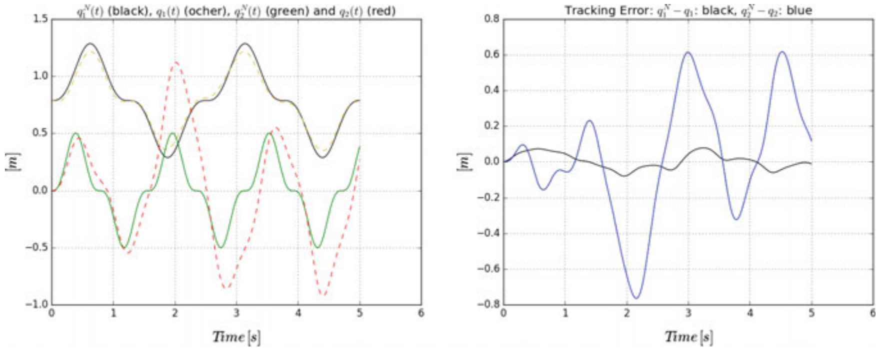


Fig. 7 Trajectory tracking of the non-adaptive controller for the "*soft computing-based model*"

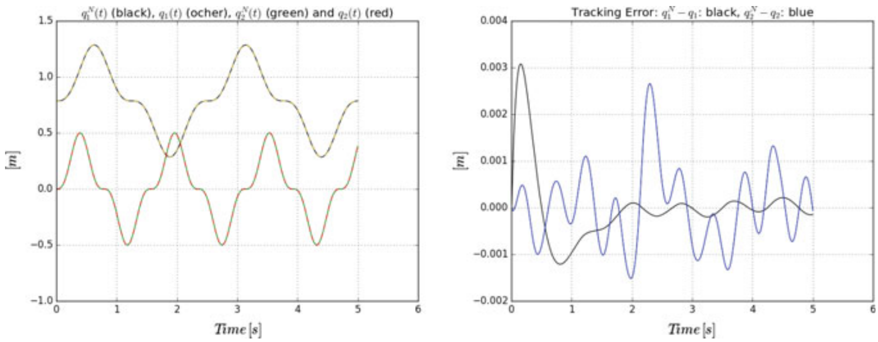


Fig. 8 Trajectory tracking of the adaptive controller for the "*soft computing-based model*"

```
    H=zeros(2,2)
    sq1=mus(q[1,1])
    H[1,1]=ma*La^2;
    H[2,2]=ma*La^2*sq1^2;
    h=[1.0;1.0];
    return H*q_ppDes+h;
end
```

From Figs. 9 and 10 it can be seen, that the fuzzy modeling improved the precision.

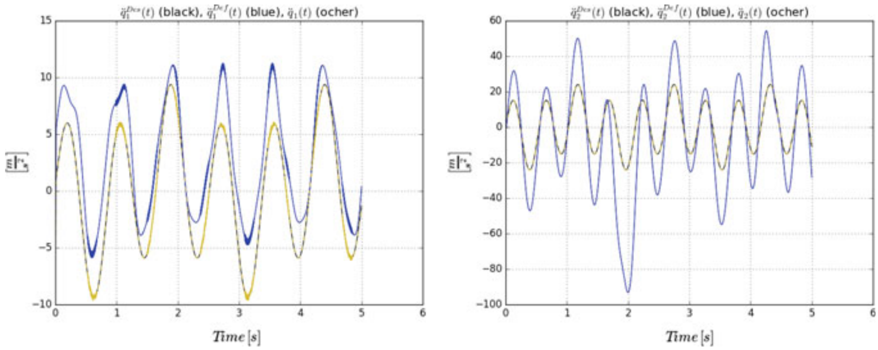


Fig. 9 The $\ddot{q}$ values of the adaptive controller for the "*soft computing-based model*"

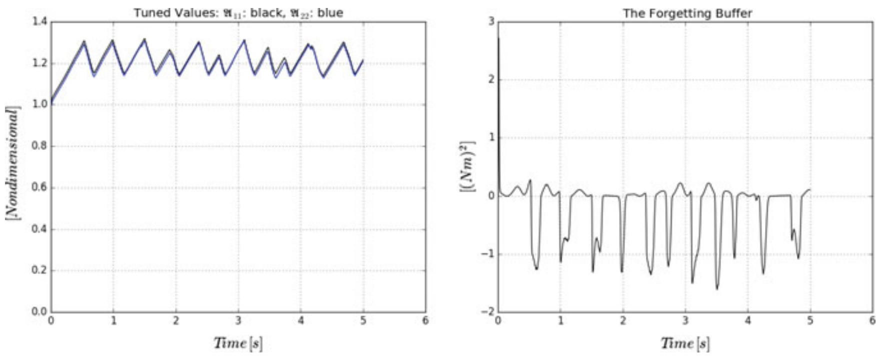


Fig. 10 The tuned parameters of the adaptive controller and the content of the forgetting buffer for the "*soft computing-based model*"

3.3 Performance Using the Fully Soft Computing-Based Model

In the case of the *fully soft computing-based model* in the *approximate dynamic model* each term of the functions $\sin(x)$ and $\cos(x)$ were approximated by the fuzzy $\mu_s(x)$ and $\mu_c(x)$ functions according to the code below:

```
function ApprMod(q,q_p,q_ppDes)
    global ma
    global La
    global ga
    H=zeros(2,2)
    sq1=mus(q[1,1])
    cq1=muc(q[1,1])
    H[1,1]=ma*La^2;
```

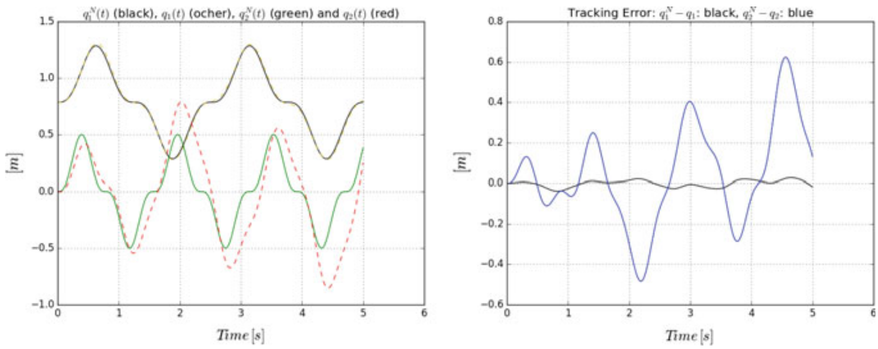


Fig. 11 Trajectory tracking of the non-adaptive controller for the "*soft computing-based model*"

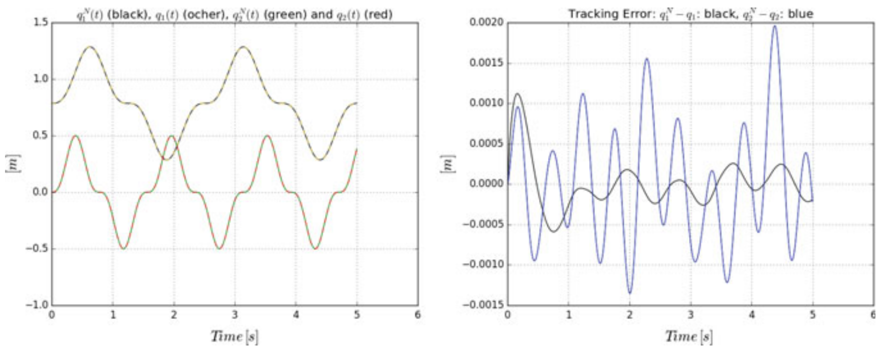


Fig. 12 Trajectory tracking of the adaptive controller for the "*fully soft computing-based model*"

```
    H[2,2]=ma*La^2*sq1^2;
    h=[1.0;1.0];
    h[1,1]=-ma*La^2*sq1*cq1*q_p[2,1]^2+
    +ma*ga*La*sq1;
    h[2,1]=2*ma*La^2*sq1*cq1*q_p[1,1]*q_p[2,1];
    return H*q_ppDes+h;
end
```

The results revealed by Figs. 11, 12, 13 and 14 validate, that by the combination of fuzzy modeling and the *Sigmoid Generated Fixed Point Transformation* design can ensure high performance and robustness to modeling imprecisions.

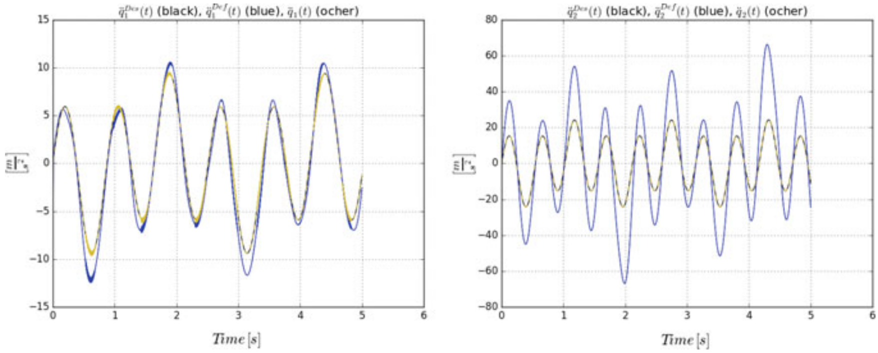


Fig. 13 The $\ddot{q}$ values of the adaptive controller for the "*fully soft computing-based model*"

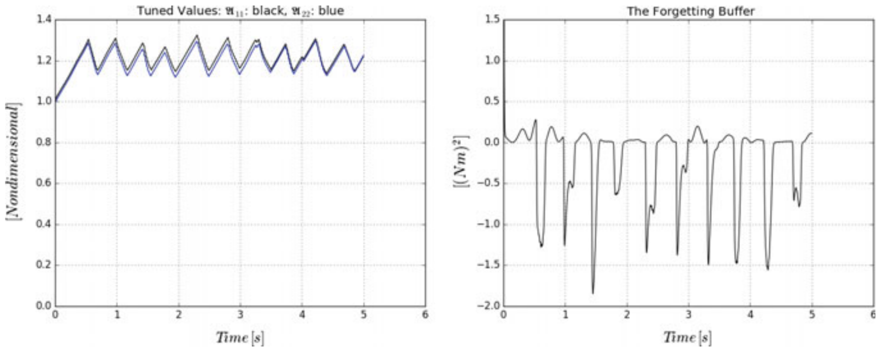


Fig. 14 The tuned parameters of the adaptive controller and the content of the forgetting buffer for the "*fully soft computing-based model*"

4 Conclusions

We have presented a new adaptive control method by the combination of fuzzy modeling and the *Sigmoid Generated Fixed Point Transformation*. Recent years, the "*Sigmoid Generated Fixed Point Transformation (SGFPT)*" has been introduced that can serve as an efficient alternative of the Lyapunov technique in nonlinear control design. This systematic method has been proposed for the generation of whole families of Fixed Point Transformations. In addition it has been extended from Single Input Single Output (SISO) to Multiple Input Multiple Output (MIMO) systems. Many studies regarding modeling issues highlight the efficiency of soft computing methods. In order to handle errors occurring due to modeling imprecisions we have introduced fuzzy approximation in this technique. For the numerical simulations the inverted pendulum system served as a benchmark problem. The "*affine*" model has been investigated in comparison with the "*soft computing-based*" model. In the soft computing-based model the trigonometric terms of the approximate dynamic model were replaced by fuzzy rules. In the case of the "*fully soft computing-based*" model each term the functions sin(x) and cos(x) were approximated by the fuzzy rules. Additionally, the SGFPT type control has been compared to a non-adaptive one. The results have confirmed that by applying fuzzy modeling in the SGFPT control design the performance level can be significantly increased. Taken together, these investigations confirm the usability and efficiency of this new method.

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